Prof. Dr. Bernd Dreseler Sommersemester 2020

Übungen zur Harmonischen Analyse

Aufgabe 1 Prove that

$$u(\phi) = \phi^{(k)}(0) \qquad (\phi \in C_0^\infty(\mathbf{R}))$$

defines a distribution $u \in \mathcal{D}'(\mathbf{R})$ of order $\leq k$.

Aufgabe 2 Show that the distribution u from the preceding problem is not of order strictly lower than k in any neighborhood of 0. Hint: examine $u(\phi)$ and $\|\phi\|_{C^{k-1}}$ forϕ $= \phi_\epsilon$, where $\epsilon > 0$ and $\phi_\epsilon(x) = x^k \chi(x/\epsilon)$ with $\chi \in C^\infty(\mathbf{R})$ and χ equals 1 on a neighborhood of 0. To calculate $u(\phi_\epsilon)$ and $\|\phi_\epsilon\|_{C^{k-1}}$, use Leibniz' formula (2.7) in the case $n = 1$.

Aufgabe 3 We consider functions and distributions on $\mathbf{R}^2$.

~~(i)~~ $r(x) = \|x\|$ defines a function on $\mathbf{R}^2$.

(a) Verify that $\log r$ and $1/r$ define distributions on $\mathbf{R}^2$. What is the order of these distributions?

(b) How about $1/r^2$?