Übungsblatt 4

Funktionalanalysis WS 2021/22

08.11.2021

1. Let $1 \leq p < oo$ and $G_p = \left\{ (x_n)_{n \in \mathbb{N}} \in l_p \,\middle|\, \sum_{n=1}^{\infty} x_n = 0 \right\}$.

i) Prove that $G_p \subseteq l_p$ is a linear subspace.
ii) Is $G_p \subseteq l_p$ a closed set?

2. Let X be a normed space. Find all the linear subspaces $L \subseteq X$ which contain a ball.

3. Let X and Y be two normed spaces, $T : X \to Y$ is a linear operator and $T_n : X \to Y$, $n \in \mathbb{N}$, is a sequence of linear operators. Prove that the sets

$$A = \{x \in X \mid T_n x \text{ does not converge towards } Tx\}$$

and

$$B = \{x \in X \mid (T_n x)_{n \in \mathbb{N}} \text{ is not a Cauchy sequence}\}$$

are either empty, or dense in X.

4. Let X be a normed space and $A \subseteq X$ a set with the property that $X \backslash A$ is a linear subspace. Prove that A is either dense, or empty.

5. Let X be a normed space and $G \subseteq X$ a linear subspace. Prove that either $G = X$, or $\overset{\circ}{G} = \varnothing$.