Funktionalanalysis WS 2021/22

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7. Let $(\xi_n)_{n\geq 0} \in l_\infty$. We define $U : l_1 \to C[0,1]$, $(Ua)(x) = \sum_{n=0}^{\infty} a_n \xi_n x^n \ \forall x \in [0,1]$, $\forall a \overset{X}{=} (a_n)_{n\geq 0} \in l_1$. Prove that U is well defined, linear, and continuous, with $\|U\| = \sup_{n\geq 0} |\xi_n|$.

8. Let $1 \leq p < \infty$ and $U : l_\infty \to L_p[0,1]$, $U(x_1, x_2, ...) = \sum_{n=1}^{\infty} x_n \chi_{[\frac{1}{2^n}, \frac{1}{2^{n-1}})}$. Prove that U is a linear and continuous operator, and calculate $\|U\|$.

9. We consider the linear operators $A_n : l_2 \to l_2$, $B_n : l_2 \to l_2$, $A_n(x) = (x_1/n, , x_2/(2n), x_3/(3n), ...)$, $B_n(x) = (0, 0, ..., 0, x_{n+1}, x_{n+2}, ...) \ \forall x = (x_1, x_2, ...) \in l_2$, $\forall n \in \mathbb{N}$. Prove that $\|A_n\| \to 0$ and $B_n(x) \to 0 \ \forall x \in l_2$, but that $(B_n)_{n\in\mathbb{N}}$ does not converge towards 0 in the operator norm.

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10. Let X and Y be two normed spaces such that $X \neq \{0\}$. If $L(X,Y)$ is a Banach space, prove that Y is a Banach space.

Korollar III.1.6 *In jedem normierten Raum X existiert zu jedem $x \in X$, $x \neq 0$, ein Funktional $x' \in X'$ mit*

$$\|x'\| = 1 \quad \text{und} \quad x'(x) = \|x\|.$$

Speziell trennt X' die Punkte von X; d. h., zu $x_1, x_2 \in X$, $x_1 \neq x_2$, existiert $x' \in X'$ mit $x'(x_1) \neq x'(x_2)$.

☐Benutzen!!