Übungsblatt 9

Funktionalanalysis WS 2021

06.12.2021

1. i) Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of scalars with the property $\forall\,(x_n)_{n\in\mathbb{N}} \in c_0$ it follows that $(a_n x_n)_{n\in\mathbb{N}} \in c_0$. Prove that $(a_n)_{n\in\mathbb{N}} \in l_\infty$.

ii) Let $(a_n)_{n\in\mathbb{N}}$ be a sequence of scalars with the property $\forall\,(x_n)_{n\in\mathbb{N}} \in l_\infty$ it follows that $(a_n x_n)_{n\in\mathbb{N}} \in c_0$. Prove that $(a_n)_{n\in\mathbb{N}} \in c_0$.

2. Let Y be a Banach space, Z a normed space, and $B_n : Y \to Z, n \in \mathbb{N}$, a sequence of linear and continuous operators with the property $\forall (y_n)_{n\in\mathbb{N}} \subseteq Y$ with $\|y_n\| \to 0$, it follows that $\|B_n(y_n)\| \to 0$. Prove that $\sup\limits_{n\in\mathbb{N}} \|B_n\| < \infty$.

3.

Sei E ein Banach-Raum, $(A_n), (B_n)$ Folgen in $L(E)$, sowie $A, B \in L(E)$. Es gelte

$$A_n \longrightarrow A,\ B_n \longrightarrow B \qquad \text{punktweise } (n \longrightarrow \infty).$$

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$$A_n B_n \longrightarrow AB \qquad \text{punktweise } (n \longrightarrow \infty).$$